

Peer Tutoring in Pure Mathematics Subject

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Abstract

Students need to be endowed with generic skills so as to meet the challenges of an ever changing and knowledge-based information society like Hong Kong. Traditional education systems that focus on memorization and examination cannot satisfy these new demands. So there is a need for the shift of learning and teaching (L&T). This paper shares the experience of launching “Peer Tutoring” in Advanced-Level Pure Mathematics subject in one school. It focuses on how peer tutoring influences students’ learning behaviors and attitudes, learning effectiveness and skill promotion. The change of roles for students and teachers, advantages and limitations of peer tutoring will also be discussed.

Keywords

Peer tutoring, Learning behavior, Learning attitudes, Learning effectiveness, Skill promotion

INTRODUCTION

Educational reform in an information society like Hong Kong in the 21st century focuses on equipping students with self-learning skills and broadened knowledge. According to the Curriculum Development Council, the paradigm of “imparting knowledge” is shifting to that of “fostering students to learn how to learn”, which involves reform of roles of students and teachers in the processes of learning and teaching (L&T). It is important to teach students generic skills in order to help them learn how to learn. These generic skills namely

communication, critical thinking, creativity, collaboration, information technology, numeracy, problem solving, self management and study skills, should be developed through L&T in the contexts of different subjects. Hence, teachers should no longer be merely knowledge transmitters, but also facilitators of the independent learning process of their students (Curriculum Development Council, 2001). In response to the above-mentioned reform, peer tutoring can be a cultural change in L&T.

WHAT IS PEER TUTORING?

“Peers” is defined as someone belonging to the same group, in terms of status, in society. Here “peers” refers to the students in the same class taking the same subject. Peer tutoring involves one or more students teaching other students in a particular subject area. This provides one-on-one attention, immediate feedback, and active “discovery learning”, which are especially effective in science education, in a non-threatening environment. Being a mathematics teacher, I have been attracted to catering students’ individual difference using “peer tutoring” in my lesson plan. Peer tutoring seems to be a good solution to help minimize individual differences.

Theoretical Bases of Peer Tutoring

According to the basic cognitive developmental theories of Piaget and Vygotsky, human development can be viewed as constructive or socio-genetic processes. In the views of Piaget’s constructivism, “human beings are capable of extending biological programming to construct cognitive systems that interpret experiences with objects and other persons...Peer interactions provide rich and necessary contexts for students to revise their current cognitive system. Such revisions would, in turn, lead students to make new meanings.” (O’Donnell and King, 1999, p.5) Vygotsky’s theory views human development as a socio-genetic process by which children gain cognitive growth by interacting with others who are often more competent. Such process will be effective if the interaction occurs within one’s “zone of proximal development”. Instead of being a passive learner, students should be encouraged to have more interaction with others.

It is known that verbal interaction is important for cognitive change and knowledge construction (Forman and Cazden, 1985), and students would have better cognitive growth if they were working with each

other than working alone (Bell et. al, 1985). Students can also learn skills, beliefs and behaviors by imitating peers and adults without interacting with them (Bandura, 1986). They can figure out their misconceptions and thus correct them by observing others (models). A number of studies supports that modeling can promote subsequent performance on the observed task (e.g. Kuhn, 1972; Murray, 1972; Zimmerman, 1974). However, modeling is ineffective when models function far in advance of the participants (Kuhn, 1972). Hence, a peer model is better than an adult model because students, observing a peer completing a task successfully, would probably increase their self-concept and believe that they could also succeed in the same task.

On the other hand, many researchers find that teenagers inhabit cultural worlds far different from those of their teachers (e.g. Murdock and Phelps, 1973). As students communicate at the same levels of knowledge background and use the same language, they are more likely to accept one and others’ opinions (Piaget, 1976) and are generally more willing to challenge each other than to challenge teachers. Besides, compared with those, like most of the teachers, who had had no difficulties in learning, students who had struggled themselves to understand a concept may be advantaged in providing “scaffolding” - the process of enabling one to carry out a task that he /she would not be able to perform without assistance. It is because peer students are more sensitive than teachers to non-verbal cues given by other students to indicate that they do not understand something (Allen, 1976), and they are better able to explain concepts in much familiar terms (Noddings, 1985). Thus, students will learn better from tutors who are their peers, or who are similar in general culture and background, than from teachers.

Most students find that Advanced-level Pure Mathematics is, to a certain extent, quite abstract, and

so do my students. They always reflect that after observing my solving some mathematical problems, they can learn new skills in tackling similar problems, but this may not necessarily enhance their problem solving confidence. I have tried to launch peer tutoring in teaching some of the topics, aiming at strengthening students' problem solving confidence.

General Benefits to Participants in Peer Tutoring Scheme

Benefits to Tutees

According to Doise et. al (1975), when children are presented with a solution that contradicted their own, even if the contradiction is not a correct one, will promote children's progress. In peer tutoring, students are exposed to a more challenging mode of class interaction. As interaction may produce cognitive conflicts and disagreement among students, to resolve, students have to discuss, understand, try and accept new ideas. Such a process makes students to reflect, justify and explain their own beliefs and thus help students to learn through seeking and comprehending alternate solutions. Student tutees gain from the process of understanding others' ideas, correcting previous misconceptions and filling the gaps between prior knowledge and new ideas. Finally, students work together to invent their own problem-solving procedures and discover their own solutions, creating an atmosphere of social stimulation and support (Damon and Phelps, 1989), resulting in enhancing students' communication and collaboration skills.

As a matter of fact, many research findings showed that peer tutoring produced significantly greater achievement than normal classroom instruction (e.g. Bausell, Moody and Walzl, 1972; Russell and Ford, 1983). Allen and Feldman (1973) found that children learned better by peer tutoring in science-related topics

than by studying alone. Horan et. al (1974) and Mevarech (1985) also showed that pupils who are tutored in mathematics improve mathematics achievement and classroom behavior more than control groups who are not tutored.

Benefits to Tutors

When student tutors are temporarily adopting the teacher's role, their behavior will be constrained by the expectations of peer students, leads to develop sympathy with their teachers and began to cooperate with them. This results in improved behavior in tutors' own classrooms, a better attitude towards schoolwork and a deeper respect for learning (Geiser, 1969; Allen, 1976). Tutors will also learn responsibility, caring for others (Coleman et. al, 1974), gain the experience of being needed, valued, and respected. Such an experience produces a new view of self as a worthwhile human being (Hedin, 1987), develops tutors' sense of personal adequacy, enhances their feeling of self-esteem, and improves their self-concept (Mainiero et. al, 1971).

In order to convince others, students who teach others have to struggle to make the material meaningful to the learners, thereby have the opportunity of reflecting upon their own learning process and reinforcing tutors' knowledge of fundamentals (Bruner, 1963). In the process of giving explanations and justifying their ideas to student tutees, student tutors may review and restructure the knowledge they possess, clarify or recognize material in new ways in their own mind, recognize and fill in gaps in their understanding to see how their subject area "all fits together", identify and resolve inconsistencies, and construct more elaborate conceptualizations (Yachel et. al, 1991). Student tutors then could be benefited from the process of reorganizing and restructuring their thinking for nothing clarifies their ideas as much as explaining them to others. In fact, the

tutor receives the most gains in peer tutoring scheme. What is more, Goodlad and Hirst (1989) suggested peer tutoring could give tutors a chance to make direct use of the knowledge they already possessed and might, consequently, inspire them to seek more of it.

Benefits to Teachers

As mentioned previously, students who are mobilized with the role of a tutor will sympathize their teachers and improve their classroom behavior, this may reduce discipline problems and create an atmosphere of cooperation that make teacher's job pleasanter. Peer tutoring can also free teachers from routine tasks, giving them more time on planning the curriculum and arranging conditions in which students can learn (Goodlad and Hirst, 1989).

Furthermore, according to a Stanford University study, peer tutoring is consistently more cost-effective than computer-assisted instruction, reduction of class sizes or increased instructional time for raising both reading and mathematics achievement of both tutors and tutees (Levin, 1984), which means, teachers can use their resources more effectively.

IMPLEMENTATION OF PEER TUTORING

Target Students

All Matriculation (Form 6-7) Mathematics Group students in Stewards Pooi Tun Secondary School, sitting for the 2003 Hong Kong Advanced Level (HKAL) Pure Mathematics Examination participated in the peer-tutoring scheme. There were 15 students, with average grades of about B and C respectively in the Hong Kong Certificate (HKCE) General Mathematics and Additional Mathematics Examinations (2001).

Duration of the Peer Tutoring Scheme

In order to keep pace with the teaching schedule, the peer-tutoring scheme was first launched as a pilot in the term break period (Form 6) from mid-June to August in 2002, when both the teacher and the students were less busy and were more capable of accepting such a challenging shift of mode of learning. The scheme was taken place in the post-examination classes (before the summer vacation started) and the summer tutorial classes, twice a week in about two-month time. Each class lasted for three hours. After that, students were encouraged to use peer tutoring regularly in the new semester (Form 7).

Topics included in the Peer-tutoring Scheme

As the students had to adopt the teaching role, topics included in peer tutoring should not be too difficult for them. Two topics, namely "Indefinite Integrals" and "Definite Integrals", were chosen in the pilot classes because all the students had already acquired the basic knowledge and skills in tackling problems involving "integration" when they were in Form 5. The teacher first tailored the chosen topics into several sub-topics. Then divided the class into five groups, each of three students. The groups were allowed to choose the sub-topics on their own free will. Each group prepared their own teaching material or lecture notes and took turn to deliver lectures. For the audience, they had to take an active part in class discussion. Later in the new semester (Form 7), instead of delivering lectures, students were only asked to use peer tutoring regularly in solving mathematics problems, especially those in the past HKALE papers.

Teacher's Role in Peer Tutoring

In launching peer tutoring, in which students shared teacher's duty in delivering lectures doesn't mean freeing teacher from lesson preparation. Instead, it shifts teacher's role in lesson design. Teacher's new roles are mainly focused on the following:

1. Preparations

Teacher should help students to form groups, considering factors like gender, abilities and pre-existing social relationship of students so as to maximize their performance. He/she then breaks down the teaching materials into a series of digestible snippets, and helps to set clear objectives and goals. It is also necessary to provide material needed in presentation and help to set preparation schedule to make sure that student tutors have sufficient preparation time.

2. During Students' Presentation

Teacher should be responsible for monitoring and supervising students' presentation, offering instantaneous feedback like giving praises and comments, correcting mistakes, strengthening and reorganizing key points. Besides, teacher should engage and challenge students to arouse discussion, and teach critical thinking and questioning skills.

3. Lesson Enrichment

Teacher is also expected to provide intellectual inspiration and leadership to arouse interest, and finally, to focus on teaching new or supplementary information.

EVALUATION ON THE EFFECTIVENESS OF PEER TUTORING

In order to assess the effectiveness of peer tutoring, students' perception of their dispositions and outcomes in learning were measured for both the "teacher-as-instructor" and the "student-as-instructor" modes of study. The main questions investigated in this study were, "Did the students benefit from the change in modes of study towards student-as-instructor learning style?" and "What are the advantages and limitations of peer tutoring?"

The Instrument

A self-developed questionnaire was introduced to measure four subscales of students' dispositions and outcomes in learning as listed below:

1. Learning Behaviors - it measured students' perception of their behavior in learning like how concentrated and actively involved during class discussions, and how well they collaborated in learning.
2. Learning Attitudes - it measured students' perception of their disposition in the dimension of motivation, confidence in problem-solving, self-concept, and how challenging they found the lessons.
3. Learning Effectiveness - it measured students' perception of their disposition in the dimension of

effectiveness, learning climate, understanding of concepts, and catering individual difference.

4. Skill Promotion - it measured students' perception of their disposition in the dimension of independence and autonomy in learning, mind broadening, critical thinking skills and organization.

All the 15 Pure Mathematics students sitting for 2003 HKALE were asked to rate the items of the questionnaire on a six-point Likert scale, ranging from 1 to 6, in which 1 indicating "strongly disagree" and 6 indicating "strongly agree". In addition to the item-rating questions, two open-ended questions concerning students' opinion of the advantages and limitations of peer tutoring were also introduced in the questionnaire.

Finally, students' HKALE result, in terms of passing rate and value-added index in Pure Mathematics, will be analyzed in reviewing the overall effectiveness of the scheme.

Results and Discussions

Before going on to the data analysis, there are two important things to point out: first, since the sample is non-random and convenient with only 15 subjects, the results might not be statistically reliable nor significant, however, they still reflected something to a certain extent; second, 5 out of the 15 subjects (33%) had attended private tutorials, therefore their overall HKALE results cannot be simply attributed to the peer-tutoring scheme.

Students' Overall Perception of Peer Tutoring Scheme

Students' perception of their dispositions in the four subscales are summarized in the following:

Table 1 Means of the subscales of the questionnaire

Subscales	No of cases	Mean	
		Teacher-as-instructor	Student-as-instructor
Learning behaviors	15	4.42	4.55
Learning attitudes	15	3.97	4.25
Learning effectiveness	15	4.55	3.87
Skill promotion	15	4.14	4.17

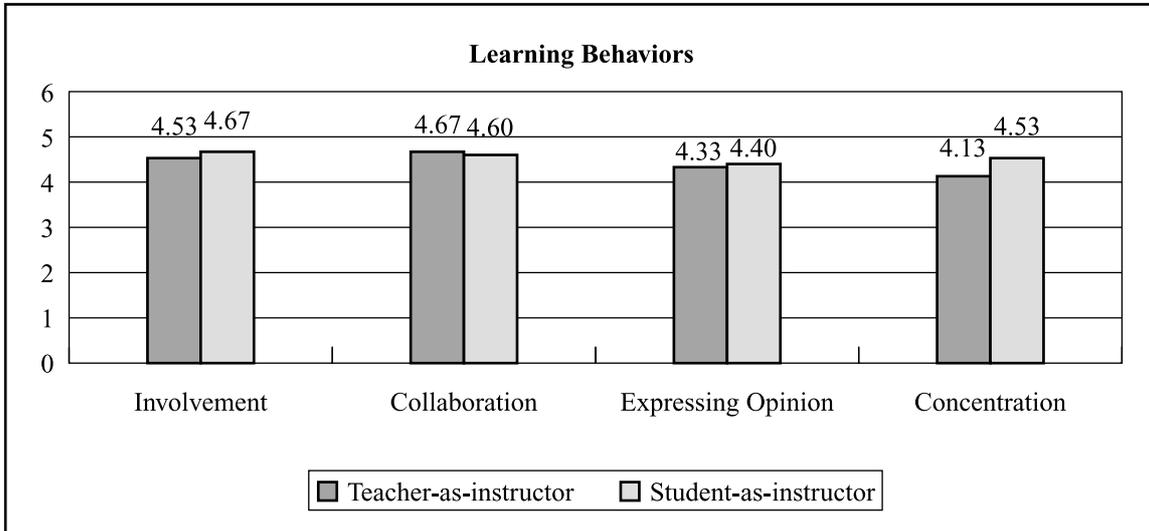
Results from the questionnaire indicate that students perceived that the "learning attitudes" in the student-as-instructor mode of study was higher than that in the teacher-as-instructor mode of study by 0.28, while

their "learning behaviors" and "skill promotion" were just slightly better in peer tutoring. Interestingly, they reflected that peer tutoring was far less effective than conventional classroom instruction by 0.68.

Let's go into details of each of the subscales:

1. Learning Behaviors:

Table 2 Bar Chart indicating the Means of Sample Items of the Subscale: Learning Behaviors

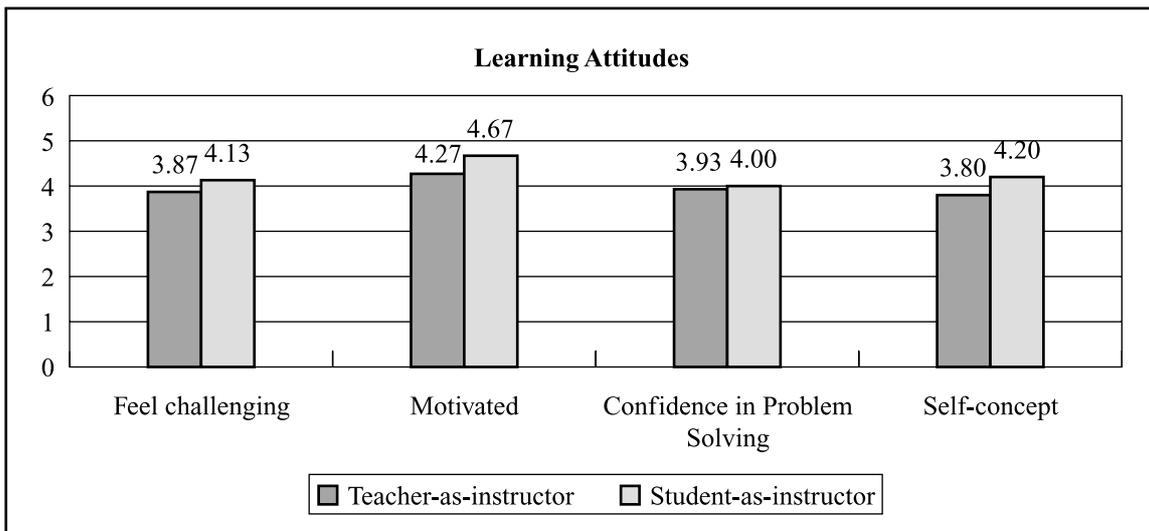


Students showed greater “involvement” and had more “collaboration” in peer tutoring, but they didn’t find much difference in terms of “concentration” and

chances of “expressing opinion” in both modes of learning.

2. Learning Attitudes:

Table 3 Bar Chart indicating the Means of Sample Items of the Subscale: Learning Attitudes

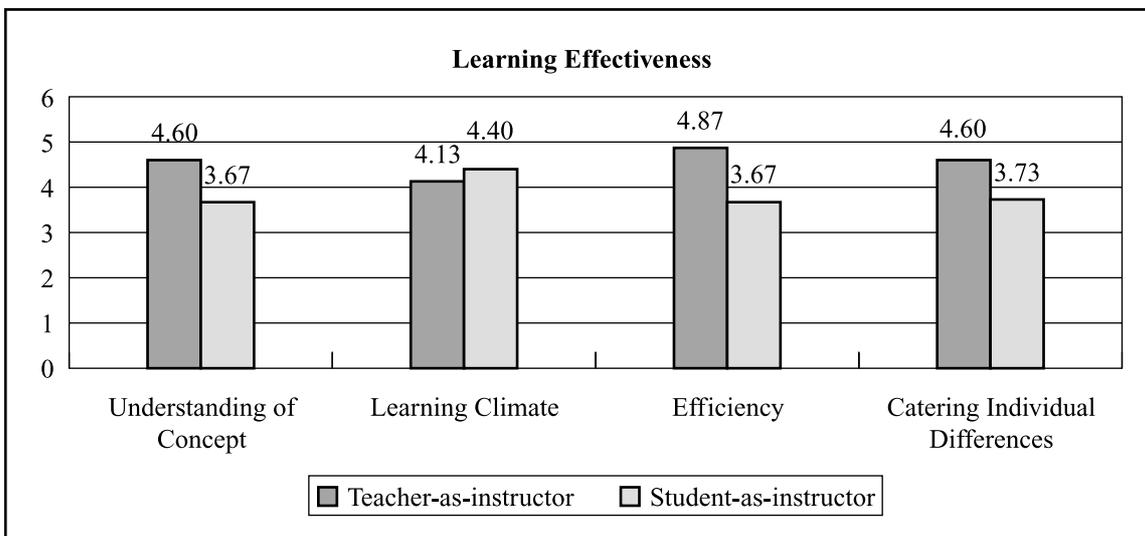


Students found that peer tutoring was much more “challenging”. It was better able to “motivate” their studies and improve their “self-concept”. However,

watching their peer tutors solving various problems successfully might not necessarily upgrade their “confidence in problem-solving”.

3. Learning Effectiveness:

Table 4 Bar Chart indicating the Means of Sample Items of the Subscale: Learning Effectiveness

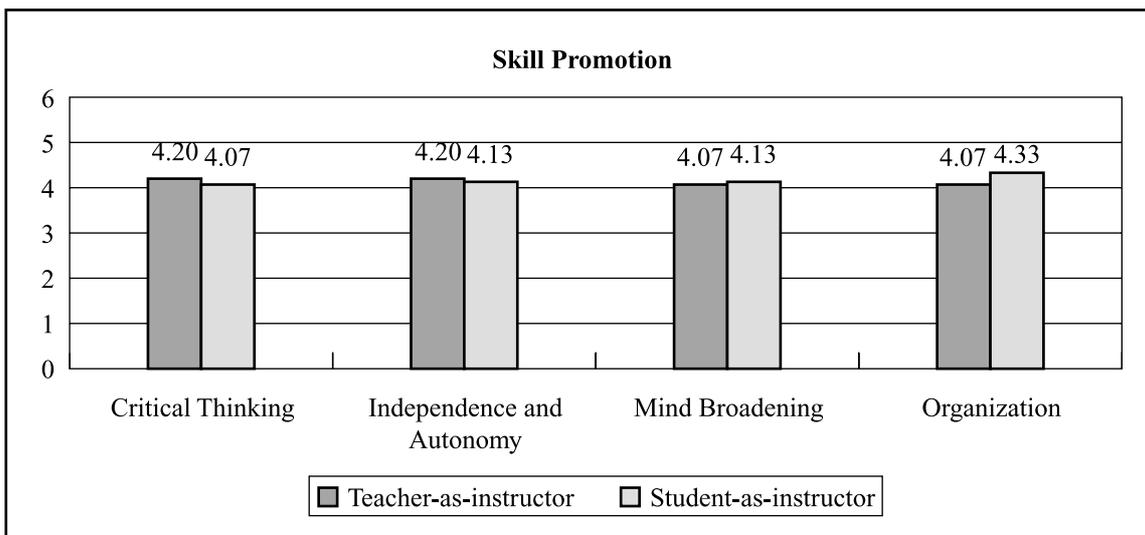


Though students agreed that peer tutoring could enhance “learning climate” in class, they pointed out that it was rather time-consuming, and that’s why they rated “efficiency” in the student-as-instructor mode far lower than that in the teacher-as-instructor mode by 1.20. They

too strongly reflected that with pedagogic training and rich teaching experience, teacher was more professional and experienced than peer tutors in “catering individual differences” and explaining “concepts”, especially those abstract ones.

4. Skill Promotion:

Table 5 Bar Chart indicating the Means of Sample Items of the Subscale: Skill Promotion



The results indicate that in peer tutoring, students would have better growth in “organization”. Contradicting to the research review, students showed that they learnt “critical thinking” skill more in the teacher-as-instructor mode of learning. This might be attributed to the fact

that the teacher used to prepare teaching materials intended for training students’ critical thinking. For example, the teacher often presented examples of different problem-solving methods, some were even conceptually wrong, and asked students to criticize or modify them.

Result Summary

Most of students, taking the role as student tutees, agreed that they were more motivated and actively involved in class for they had to listen extensively and attentively because, unlike in the “teacher-as-instructor” mode of study, they could not take for granted that whatever student tutors taught were all correct. They tended to be more critical-minded and learned to challenge their peer tutors, making the lessons more challenging and thus promoting learning climate. Besides, through peer tutoring, students had more opportunities to collaborate and organize their learning material and to share with their peer, meaning that every one could contribute to the learning outcomes of the whole class, resulting in an increase in self-concept and organization skills. In spite of this, students found that peer tutoring was less efficient, in terms of understanding new concepts and catering individual differences.

Students’ Opinions in the Two Open-ended Questions

1. What did you like best about peer tutoring?

- *“In order to participate in class discussion, I had to prepare for the lessons. This helped me to build a habit of reading teaching materials before class.”*
- *“We had to prepare lessons in groups, this helped us to develop learning bonds with student peers.”*
- *“During peer tutors’ presentation, I was less likely to accept all what they presented, this sometimes aroused conflicts among us. To resolve, I was encouraged to develop a tolerance for uncertainty and conflict.”*
- *“The learning climate was much more challenging, and consequently increased my motivation, concentration and involvement in class.”*

- *“As all of us had the opportunities to teach and to challenge our peers, we could make significant contribution to every student’s knowledge construction. This not only increased our self-esteem, self-confidence and leadership, but also built up our spontaneity in learning.”*
- *“Peer tutoring helped us to move away from dependence on teachers’ authority toward belief in our own ability to seek knowledge and figure out solutions.”*
- *“Peer tutoring promotes cooperation, friendliness, positive social behavior like giving praise and encouragement, and hence improve our communication and interpersonal skills.”*

2. What are the limitations in peer tutoring?

- *“Without pedagogic training, students able to understand a concept well does not necessarily imply that they can teach the concept equally well.”*
- *“We may have to pay extra effort in adapting ourselves to the different teaching styles of different student tutors.”*
- *“It may cause disciplinary problem if the students are not mature and disciplined enough.”*
- *“Peer tutoring is in overall much more time-consuming than traditional classroom teaching. This may cause difficulty in keeping pace with the teaching schedule, especially when there is an existing syllabus for public examination.”*
- *“We may learn wrong concepts if the teacher is not alert enough in correcting mistakes made by the student tutors immediately.”*
- *“Not every topic is suitable to implement peer tutoring. Some topics may be too abstract and require much of teacher’s elaboration, or some may be too hard to break down into a series of digestible snippets.”*

Students' Hong Kong Advanced-Level Examination Result in Pure Mathematics (2003)

The passing rate of the fifteen students in 2003 HKAL Pure Mathematics Examination was 93.3%. Three students (20%) got “distinctions”; all these three students were the most active ones involved in peer tutoring and only one out of them had attended private tutorials.

According to the Value-Added index released by the Education and Manpower Bureau (Table 6), the target students had an overall estimate of 5.71 value added. Since factors affecting learning outcomes are too complex, we cannot simply attribute the value added to the launching of peer-tutoring scheme. Nevertheless, peer tutoring indeed created a self-learning atmosphere and promoted students' generic skills that are essentials of effective learning.

Table 6 Value-Added Index of 2003 HKAL Pure Mathematics Examination

2003	Value-Added			Stanine
	Low	Estimate	High	
Pure Math	2.2	5.71	8.86	8
Ref. Range	-9 to 9			1 to 9

CONCLUSION AND SUGGESTIONS

Hong Kong is now developing towards an information society in which students need to be endowed with generic skills so as to meet the challenges of such an ever changing and knowledge-based environment. Traditional classroom's board-and-chalk teaching that focuses on memorization and examination can merely contribute to these new requirements. So there is an undeniable need for the shift of teaching mode. Peer tutoring provides an alternative for both learning and teaching. It helps to upgrade students' learning behaviors and attitudes, enhances critical thinking and organization skills, and promotes communication and collaboration. These lead to a self learning climate in which students move away from the dependence on teachers' authority toward self-confidence in problem solving.

In order to yield greater benefits from peer tutoring, it is suggested that as the pre-requisite for both

the student tutors and tutees, they should be considerably disciplined and have to be taught some bases on certain topics before they can teach. For the topics chosen, they should not be too abstract nor far beyond students' grasp, because presenting students with difficult problems will not necessarily result in productive discussion and cognitive change. Instead, a decline in correct responding and more deeply entrenched misunderstanding may result (Levin and Druyan, 1993). Last but not the least, the teacher should be alert enough throughout student tutors' presentation so as to correct any misconceptions immediately.

Peer tutoring should be seen as one of the many teaching strategies available in education reform. It is not intended to replace other teaching techniques, but rather to complement them. It will always remain the teacher's privilege and responsibility to decide wisely which teaching method is beneficial and desirable for a given group of students and given subject matter.

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